

Solve Word Problems with Ratios, Proportions, and Rates

Facts to Know

Ratios

- A ratio is used to compare two numbers or the size of two amounts.
- A ratio can be used to compare part of something to the entire amount as you do in a fraction.
- A ratio can compare one part of a whole to another part of a whole.
- A ratio can compare all of one thing to all of something else.

Sample A

Kathy has 7 orange tennis balls and 5 yellow tennis balls.

The ratio of orange tennis balls to all tennis balls is $\frac{7}{12}$

The ratio of yellow tennis balls to all tennis balls is $\frac{5}{12}$

The ratio of orange tennis balls to yellow tennis balls is $\frac{7}{5}$

The ratio of yellow tennis balls to orange tennis balls is $\frac{5}{7}$

Writing Ratios

- A ratio can be written as a fraction: $\frac{2}{3}$
- A ratio can be expressed with a colon: 2:3.
- A ratio can be written with "to": 2 to 3.

Proportions

- A proportion is used to compare two ratios.
- A proportion is an equation which shows that two ratios are equal.
- A proportion can be written in fraction form.
 $\frac{1}{2} = \frac{5}{10}$
- A proportion can be written in colon form.
1:2 :: 5:10 (1 is to 2 as 5 is to 10)
- The outer terms (1 and 10) are called the *extremes*.
- The inner terms (2 and 5) are called the *means*.

Using Proportions

- The product of the means equals the product of the extremes.
- If you know any three of the terms, you can find the fourth.
- You can also solve a proportion by using cross products.

$$1:2 :: 5:10$$

$$2 \times 5 = 10 \text{ and } 1 \times 10 = 10$$

$$1:7 :: 4:c \quad 1 \times c = c$$

$$7 \times 4 = 28 \quad c = 28$$

$$\frac{4}{6} = \frac{8}{a} \text{ or } 4 \times a = 6 \times 8$$

$$4a = 48$$

$$a = 12$$

Rates

A rate is a special ratio whose denominator is always 1.

Examples include miles per gallon (mpg) and miles per hour (mph).

Sample B

If you travel 40 miles in one hour, how far do you travel in 3 hours?

$$\text{(miles)} \frac{40}{1} = \frac{d}{3} \quad d = 120$$

$$\text{(hours)} \frac{1}{3} \quad \text{You travel 120 miles in 3 hours.}$$

Some special formulas with rates are the following:

Rate of Speed = Distance divided by Time ($R = D/T$)

Distance = Rate of Speed multiplied by Time ($D = R \times T$)

Time = Distance divided by Rate of Speed ($T = D/R$)